

Letter to the Editor: Isosceles Dissections

I enjoyed Des MacHale's "Proof Without Words: Isosceles Dissections" in the December 2008 issue of the *MAGAZINE*. However, his third claim (that a triangle can be dissected into two isosceles triangles if and only if one of its angles is three times another or if the triangle is right angled) is incorrect. There is a third case to consider.

Consider an acute $\triangle ABC$, two of whose angles are x and $2x$ ($0 < x < 45^\circ$), as illustrated in FIGURE 1a. Such a triangle can be dissected into two isosceles triangles as follows. As in FIGURE 1b, draw the cevian CD equal in length to BC so that $\triangle BCD$ is isosceles. It now follows that $\angle ACD = x$, so that $\triangle ACD$ is also isosceles.

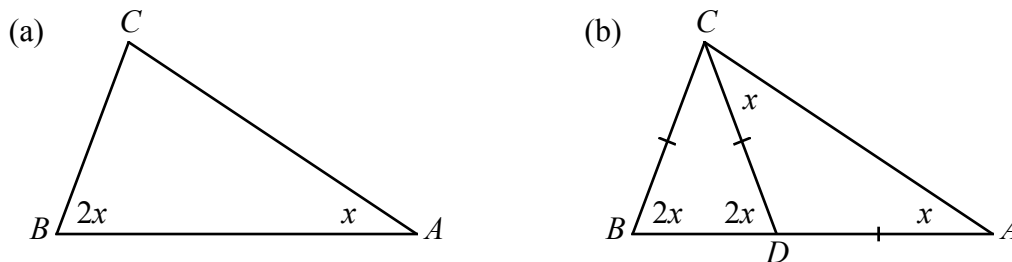


Figure 1 A triangle with one angle twice another

Two familiar members of this family are the triangles with $x = 30^\circ$ (which makes $\triangle ABC$ a right triangle) and $x = 36^\circ$ (when $\triangle ABC$ is itself isosceles). MacHale also showed how every acute triangle can be dissected into three isosceles triangles. The construction in FIGURE 1 can be extended to show that every isosceles obtuse triangle can also be dissected into three isosceles triangles, as in FIGURE 2.

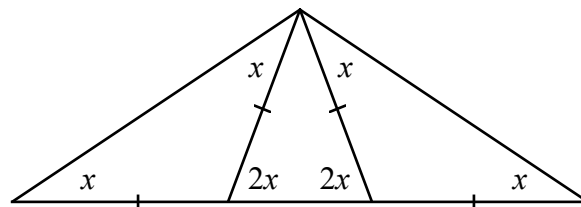


Figure 2 An obtuse isosceles triangle dissection

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